

Test for Independence

Concepts

1. To test for independence, it is just a modified version of the χ^2 test. You sum up the rows to get N_i and the columns to get M_j . Let the total sum of all the elements be S . Then, your expected distribution at square ij is $\frac{N_i M_j}{S}$, and then you perform the χ^2 test. If you have r rows and c columns, then the number of degrees of freedom is $(r-1)(c-1)$.

Examples

2. The following are the actual exit poll results from the 2016 election. Is who you vote for and your age independent?

| | 18-24 | 25-29 | 30-39 | 40-49 | 50-64 | ≥ 65 |
|---------|-------|-------|-------|-------|-------|-----------|
| Clinton | 1375 | 1194 | 2129 | 2146 | 3242 | 1768 |
| Trump | 835 | 840 | 1628 | 2286 | 3831 | 2043 |
| Other | 246 | 177 | 418 | 233 | 295 | 118 |

Problems

3. True False In the homework problem, there is a contradiction because we both keep and reject the null hypothesis that the Mendelian plants are distributed $9 : 3 : 3 : 1$.

4. You are wondering whether performing well in this course and gender are related and

you get the following table. Are they related?

| | Male | Female |
|------|------|--------|
| Pass | 175 | 725 |
| Fail | 25 | 75 |

5. You are wondering whether performing well in this course and gender are related and

you get the following table. Are they related?

| | Male | Female |
|------|------|--------|
| Pass | 315 | 485 |
| Fail | 85 | 115 |

6. Prove that the estimator for p of a geometric distribution is biased and an overestimate for sample size $n = 1$.

Miscellaneous

Examples

7. Let $v = (1, 2, 2, -1)$ and $w = (5, 3, -5, 3)$. Calculate $v \bullet w$ and $|v|$.
8. Calculate the partial derivatives of $f = 7 - x^2y^3$.

Problems

9. Find the angle between the two vector $v = (1, 3, 5, -2, 4, 3)$ and $w = (1, 1, 5, 2, 2, 1)$.
10. When is $|\vec{v} \bullet \vec{w}| = |\vec{v}| \cdot |\vec{w}|$? (Hint: What is θ ?)
11. Let $u = x^5y^4 - 3x^2y^3 + 2x^2$. Calculate u_{xx} , u_{xy} , u_{yx} , and u_{yy} .

Chi-square Distribution Table

| d.f. | .995 | .99 | .975 | .95 | .9 | .1 | .05 | .025 | .01 |
|------|-------|-------|-------|-------|-------|--------|--------|--------|--------|
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.02 | 2.71 | 3.84 | 5.02 | 6.63 |
| 2 | 0.01 | 0.02 | 0.05 | 0.10 | 0.21 | 4.61 | 5.99 | 7.38 | 9.21 |
| 3 | 0.07 | 0.11 | 0.22 | 0.35 | 0.58 | 6.25 | 7.81 | 9.35 | 11.34 |
| 4 | 0.21 | 0.30 | 0.48 | 0.71 | 1.06 | 7.78 | 9.49 | 11.14 | 13.28 |
| 5 | 0.41 | 0.55 | 0.83 | 1.15 | 1.61 | 9.24 | 11.07 | 12.83 | 15.09 |
| 6 | 0.68 | 0.87 | 1.24 | 1.64 | 2.20 | 10.64 | 12.59 | 14.45 | 16.81 |
| 7 | 0.99 | 1.24 | 1.69 | 2.17 | 2.83 | 12.02 | 14.07 | 16.01 | 18.48 |
| 8 | 1.34 | 1.65 | 2.18 | 2.73 | 3.49 | 13.36 | 15.51 | 17.53 | 20.09 |
| 9 | 1.73 | 2.09 | 2.70 | 3.33 | 4.17 | 14.68 | 16.92 | 19.02 | 21.67 |
| 10 | 2.16 | 2.56 | 3.25 | 3.94 | 4.87 | 15.99 | 18.31 | 20.48 | 23.21 |
| 11 | 2.60 | 3.05 | 3.82 | 4.57 | 5.58 | 17.28 | 19.68 | 21.92 | 24.72 |
| 12 | 3.07 | 3.57 | 4.40 | 5.23 | 6.30 | 18.55 | 21.03 | 23.34 | 26.22 |
| 13 | 3.57 | 4.11 | 5.01 | 5.89 | 7.04 | 19.81 | 22.36 | 24.74 | 27.69 |
| 14 | 4.07 | 4.66 | 5.63 | 6.57 | 7.79 | 21.06 | 23.68 | 26.12 | 29.14 |
| 15 | 4.60 | 5.23 | 6.26 | 7.26 | 8.55 | 22.31 | 25.00 | 27.49 | 30.58 |
| 16 | 5.14 | 5.81 | 6.91 | 7.96 | 9.31 | 23.54 | 26.30 | 28.85 | 32.00 |
| 17 | 5.70 | 6.41 | 7.56 | 8.67 | 10.09 | 24.77 | 27.59 | 30.19 | 33.41 |
| 18 | 6.26 | 7.01 | 8.23 | 9.39 | 10.86 | 25.99 | 28.87 | 31.53 | 34.81 |
| 19 | 6.84 | 7.63 | 8.91 | 10.12 | 11.65 | 27.20 | 30.14 | 32.85 | 36.19 |
| 20 | 7.43 | 8.26 | 9.59 | 10.85 | 12.44 | 28.41 | 31.41 | 34.17 | 37.57 |
| 22 | 8.64 | 9.54 | 10.98 | 12.34 | 14.04 | 30.81 | 33.92 | 36.78 | 40.29 |
| 24 | 9.89 | 10.86 | 12.40 | 13.85 | 15.66 | 33.20 | 36.42 | 39.36 | 42.98 |
| 26 | 11.16 | 12.20 | 13.84 | 15.38 | 17.29 | 35.56 | 38.89 | 41.92 | 45.64 |
| 28 | 12.46 | 13.56 | 15.31 | 16.93 | 18.94 | 37.92 | 41.34 | 44.46 | 48.28 |
| 30 | 13.79 | 14.95 | 16.79 | 18.49 | 20.60 | 40.26 | 43.77 | 46.98 | 50.89 |
| 32 | 15.13 | 16.36 | 18.29 | 20.07 | 22.27 | 42.58 | 46.19 | 49.48 | 53.49 |
| 34 | 16.50 | 17.79 | 19.81 | 21.66 | 23.95 | 44.90 | 48.60 | 51.97 | 56.06 |
| 38 | 19.29 | 20.69 | 22.88 | 24.88 | 27.34 | 49.51 | 53.38 | 56.90 | 61.16 |
| 42 | 22.14 | 23.65 | 26.00 | 28.14 | 30.77 | 54.09 | 58.12 | 61.78 | 66.21 |
| 46 | 25.04 | 26.66 | 29.16 | 31.44 | 34.22 | 58.64 | 62.83 | 66.62 | 71.20 |
| 50 | 27.99 | 29.71 | 32.36 | 34.76 | 37.69 | 63.17 | 67.50 | 71.42 | 76.15 |
| 55 | 31.73 | 33.57 | 36.40 | 38.96 | 42.06 | 68.80 | 73.31 | 77.38 | 82.29 |
| 60 | 35.53 | 37.48 | 40.48 | 43.19 | 46.46 | 74.40 | 79.08 | 83.30 | 88.38 |
| 65 | 39.38 | 41.44 | 44.60 | 47.45 | 50.88 | 79.97 | 84.82 | 89.18 | 94.42 |
| 70 | 43.28 | 45.44 | 48.76 | 51.74 | 55.33 | 85.53 | 90.53 | 95.02 | 100.43 |
| 75 | 47.21 | 49.48 | 52.94 | 56.05 | 59.79 | 91.06 | 96.22 | 100.84 | 106.39 |
| 80 | 51.17 | 53.54 | 57.15 | 60.39 | 64.28 | 96.58 | 101.88 | 106.63 | 112.33 |
| 85 | 55.17 | 57.63 | 61.39 | 64.75 | 68.78 | 102.08 | 107.52 | 112.39 | 118.24 |
| 90 | 59.20 | 61.75 | 65.65 | 69.13 | 73.29 | 107.57 | 113.15 | 118.14 | 124.12 |
| 95 | 63.25 | 65.90 | 69.92 | 73.52 | 77.82 | 113.04 | 118.75 | 123.86 | 129.97 |
| 100 | 67.33 | 70.06 | 74.22 | 77.93 | 82.36 | 118.50 | 124.34 | 129.56 | 135.81 |